This examination consists of two parts, A and B. Part A contains five problems of which you must select four to do. Part B contains three problems of which you must select two to do. Each problem in part A is worth 15 points and each problem in part B is worth 20 points. Only hand-in your solutions to four problems from part A and two from part B. Begin each problem on a new sheet of paper and be sure to label each page of your work with the problem number and your name. You have two hours to complete part A and one hour and 20 minutes to complete part B. There will be ten-minute break between parts A and B.

In each question, if you appeal to a theorem within your solution, you must carefully state that theorem. All graphs, unless otherwise stated, should be understood to be finite and simple.
**Problem A1:** A graph $G$ is a **chordal graph** if every cycle $C$ of $G$ contains an edge joining two non-consecutive vertices of $C$. A graph $G(V,E)$ is an **interval graph** if there is an assignment $f$ that assigns an interval $I_v$ of the real line to each vertex $v \in V(G)$ such that $uv \in E$ if and only if $I_u \cap I_v \neq \emptyset$. Prove that every interval graph is a chordal graph.

**Problem A2:** A **tournament** is an complete graph in which every edges has been given an orientation. Prove that every tournament has a directed Hamiltonian path.

**Problem A3:** Solve the recurrence

$$a_n = 5a_{n-1} - 6a_{n-2} \quad (\text{for } n \geq 2),$$

with initial conditions $a_0 = 1$ and $a_1 = 1$.

**Problem A4:** Suppose that $G$ is a connected planar graph that can be drawn in the plane so that all faces have an even number edges on their boundary. Prove that the vertices of $G$ can be properly 2-colored.

**Problem A5:** Let $h_n$ denote the number of nonnegative integral solutions of the equation:

$$x_1 + x_2 + x_3 + x_4 = n.$$

a) Write the ordinary generating function for $h_n$.

b) What is $h_{25}$?
PART B: (20 points each) Do any two. Time: 1 hour and 20 minutes.

Problem B1: Prove the given identity:

\[ \sum_{i=0}^{n} \binom{a}{i} \binom{b}{n-i} = \binom{a+b}{n} \]

b)

\[ \sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1} \]

Problem B2:

a) State the definition of what it means for a graph to be a perfect graph.

b) A graph \( G(V, E) \) is a comparability graph if there is a partial order \( P \) on \( V \) so that \( uv \in E \) if and only if \( u \) and \( v \) are comparable in \( P \). Prove that every comparability graph is perfect.

Problem B3: Use inclusion-exclusion to find a formula for the number of 1-factors in the graph obtained from \( K_{n,n} \) by removing the edges of a perfect matching.