Here are some T/F questions which may help you study for the analysis prelim.

A useful way to use this note is to answer each question and then take it apart. For example, the 3^{rd} T/F states that there is a bounded subset of $\mathbb{R}$ which has no limit points. You will easily come up with an example of this but this question should remind you of the theorem that every **infinite** bounded subset of $\mathbb{R}$ has a limit point. Also, very easy questions are mixed up with some not so easy ones. Some concepts are rehashed several times in different ways.

The Prelim Exam may have some T/F questions, some statements which need to be proved, construction of examples and counterexamples and some computational exercises.

1 True or False.

Determine whether the following statements are T/F. Give a complete justification for your answer.

1. Every open subset of $\mathbb{R}$ can be written as the pairwise disjoint union of countably many open intervals.
2. If $f : [0, 1] \to \mathbb{R}$ is continuous and $f([0, 1])$ has at least two points, then $f([0, 1])$ is uncountable.
3. There is a bounded subset of $\mathbb{R}$ which has no limit points.
4. There is an uncountable subset of $\mathbb{R}$ which has no limit points.
5. There is an uncountable subset of L^∞ which has no limit points.
6. If μ, ν are Borel measures on $\mathbb{R}$ which agree on open intervals, then they agree on every Borel set.
7. There is a nonBorel function $f : [0, 1] \to \mathbb{R}$ which is Riemann integrable.
8. If $\{f_n\}$ is a sequence of Riemann integrable functions on $[0, 1]$ which converges uniformly to some function f, then f is Riemann integrable.
9. Suppose $\{f_n\}$ is a sequence of differentiable functions on $[0, 1]$ which converges uniformly to some differentiable function, then f is differentiable.
10. Let $M \subseteq [0, 1]$ be a closed set. Then, χ_M, the characteristic function of M, is Riemann integrable.
11. There is a function which is continuous at every rational and discontinuous at every irrational.

12. $[0,1]$ is the union of a first category set and a meager set.

13. If $M \subseteq \mathbb{R}$ has positive Lebesgue measure, then M contains an interval.

14. There is a continuous piecewise monotone function $f : \mathbb{R} \to \mathbb{R}$ which is nowhere differentiable.

15. If $M \subseteq \mathbb{R}$ has Lebesgue measure zero, then M is countable.

16. If $f : [0,1] \to \mathbb{R}$ is a continuous strictly monotone function, then $\int_0^1 f' = f(1) - f(0)$.

17. $f : [0,1] \to \mathbb{R}$ is AC iff it is CBV.

18. Suppose $M \subseteq [0,1]$ is a closed set and $f : M \to \mathbb{R}$ is Lipschitz with Lipschitz constant L. Then, f can be extended to $[0,1]$ so it is Lipschitz on $[0,1]$.

19. If $f, g : [0,1] \to [0,1]$ are Lipchitz functions, then $f \circ g$ is Lipschitz.

20. The same as above when Lipschitz is replaced by AC.

21. The same as above when one is Lipschitz and the other is AC. Does it make a difference which functions goes on the outside and which goes on the inside?

22. Let $f : [0,1] \to \mathbb{R}$ be a bounded L^1 function. Then, there are 2^c many simple functions g such that $\|f - g\|_1 < .5$. Here, c represents the cardinality of continuum.

23. Given any measurable $f : [0,1] \to \mathbb{R}$, there is a measurable $g : [0,1] \to \mathbb{R}$ such that $f = g$ ae and g is continuous on a full measure subset of $[0,1]$.

24. Suppose that if $\{f_n\}$ is a sequence of Lebesgue integrable functions defined on $[0,1]$ which converges pointwise to some measurable function g. Then, there is a full measure set $A \subseteq [0,1]$ such that $\{f_n\}$ converges to g uniformly on A.

25. If M is a closed and bounded subset of a complete metric space, then M is compact.

26. Given any bounded measurable function $f : [0,1] \to \mathbb{R}$ and $\epsilon > 0$, there is a step function $g : [0,1] \to \mathbb{R}$ such that $\{x : |f(x) - g(x)| > \epsilon\}$ has measure less than ϵ.

27. Given any measurable function $f : [0,1] \to \mathbb{R}$, there is a sequence of step functions $\{f_n\}$ which converges in measure to f.

28. The same as above but convergence in measure replaced with pointwise convergence.

29. Let $M \subseteq C([0,1])$ be compact and $\{f_n\}$ be a sequence of functions from M. Then, for this $\{f_n\}$ convergence in measure is equivalent to pointwise convergence which is equivalent to uniform convergence.

30. Suppose $f_n : [0,1] \to [0,1]$ and $\{f_n\}$ converges in measure to f. Then, $\int f = \lim_{n\to\infty} \int f_n$.

31. There is a measure zero set $K \subseteq \mathbb{R}$ such that $K - K = \{x - y : x, y \in K\}$ has positive measure.

32. If $K \subseteq \mathbb{R}$ has positive measure, then $K - K$ contains an interval.

33. If $M \subseteq \mathbb{R}$ is such that M and M^c intersect every nonempty perfect subset of $\mathbb{R}$, then M is nonmeasurable.

34. There is a continuous mapping from the rationals onto some perfect subset of $\mathbb{R}$.

35. If $M \subseteq \mathbb{R}$ such that the outer measure of $[0,1] \setminus M$ is 1 minus the outer measure of M, then M is measurable.

36. L^p is separable for all $1 \leq p \leq \infty$.

37. The dual of l^∞ is l^1.

38. Every Banach space is a Hilbert space.

39. If $p \leq q$, then $L^p([0,1]) \subseteq L^q([0,1])$.

40. Suppose $T : C([0,1]) \to \mathbb{R}$ is a bounded linear map. Then, there is a signed measure μ such that $T(f) = \int f d\mu$ for all $f \in C[0,1]$.

41. l^3 has an orthonormal basis.