1. Find all homomorphisms from the group \( \mathbb{Z}_{20} \) into the group \( \mathbb{Z}_8 \).

2. Find all homomorphisms from the group \( \mathbb{Z}_4 \) into the group \( \mathbb{Z}_2 \oplus \mathbb{Z}_2 \).

3. What is the order of the factor group \( \mathbb{Z}_{60}/<15> \)?

4. What is the order of the factor group \( (\mathbb{Z}_{10} \oplus U(10))/<(2,9)> \)?

5. What is the order of the group \( U(165) \)? Express \( U(165) \) as an external direct product of cyclic additive groups of the form \( \mathbb{Z}_n \).

6. Consider the function \( \phi: \mathbb{Z}_{12} \rightarrow \mathbb{Z}_6 \) defined by \( \phi(x) = x \mod 6 \). (a) Is \( \phi \) a homomorphism? (b) What is the kernel of \( \phi \)? (c) What is the set \( \phi(\mathbb{Z}_{12}) \)? (d) What is the factor group \( \mathbb{Z}_{12}/\text{Ker}\phi \)? (e) Is \( \mathbb{Z}_{12}/\text{Ker}\phi \) isomorphic to \( \phi(\mathbb{Z}_{12}) \)?

7. Consider the mapping \( \phi: \mathbb{Z} \rightarrow 2\mathbb{Z} \) defined by \( \phi(x) = 2x \). (a) Is \( \phi \) a homomorphism? (b) What is the kernel of \( \phi \)? (c) What is the set \( \phi(\mathbb{Z}) \)? (d) What is the factor group \( \mathbb{Z}/\text{Ker}\phi \)? (e) Is \( \mathbb{Z}/\text{Ker}\phi \) isomorphic to \( \phi(\mathbb{Z}) \)?

8. Let \( \mathbb{R}^\# \) be the multiplicative group of nonzero reals. Define a function \( \theta: \mathbb{R}^\# \rightarrow \mathbb{R}^\# \) by \( \theta(x) = x^2 \). (a) Is \( \theta \) a homomorphism? (b) What is the kernel of \( \theta \)? (c) What is the set \( \theta(\mathbb{R}^\#) \)? (d) What is the factor group \( \mathbb{R}^\#/\text{Ker}\theta \)? (e) Is \( \mathbb{R}^\#/\text{Ker}\theta \) isomorphic to \( \theta(\mathbb{R}^\#) \)?

9. Let \( \mathbb{R}^\# \) be the multiplicative group of nonzero reals and \( GL(2,\mathbb{R}) \) be the matrix group of 2 by 2 matrices with real entries under matrix multiplication. Define a function \( \theta: GL(2,\mathbb{R}) \rightarrow \mathbb{R}^\# \) by \( \theta(A) = \det(A) \). (a) Is \( \theta \) a homomorphism? (b) What is the kernel of \( \theta \)? (c) What is the set \( \theta(GL(2,\mathbb{R})) \)? (d) What is the factor group \( GL(2,\mathbb{R})/\text{Ker}\theta \)? (e) Is \( GL(2,\mathbb{R})/\text{Ker}\theta \) isomorphic to \( \theta(GL(2,\mathbb{R})) \)?

10. Let \( \phi: \mathbb{Z} \rightarrow \mathbb{Z}_3 \oplus \mathbb{Z}_4 \) be a function defined by \( \phi(x) = (x \mod 3, x \mod 4) \). (a) Is \( \phi \) a homomorphism? (b) What is the kernel of \( \phi \)? (c) What is the set \( \phi(\mathbb{Z}) \)? (d) What is the factor group \( \mathbb{Z}/\text{Ker}\phi \)? (e) Is \( \mathbb{Z}/\text{Ker}\phi \) isomorphic to \( \phi(\mathbb{Z}) \)?